Butterworth Filter

420L Final Project

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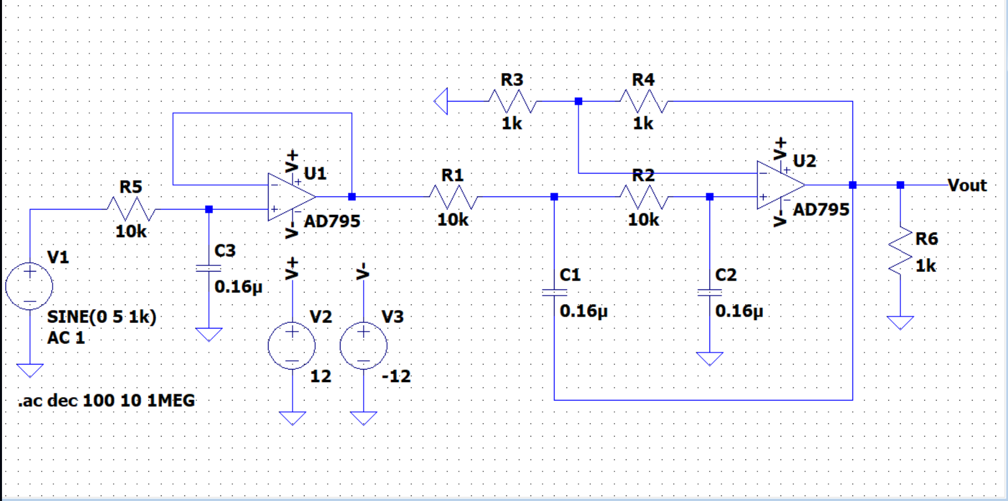
Background – Butterworth Filters

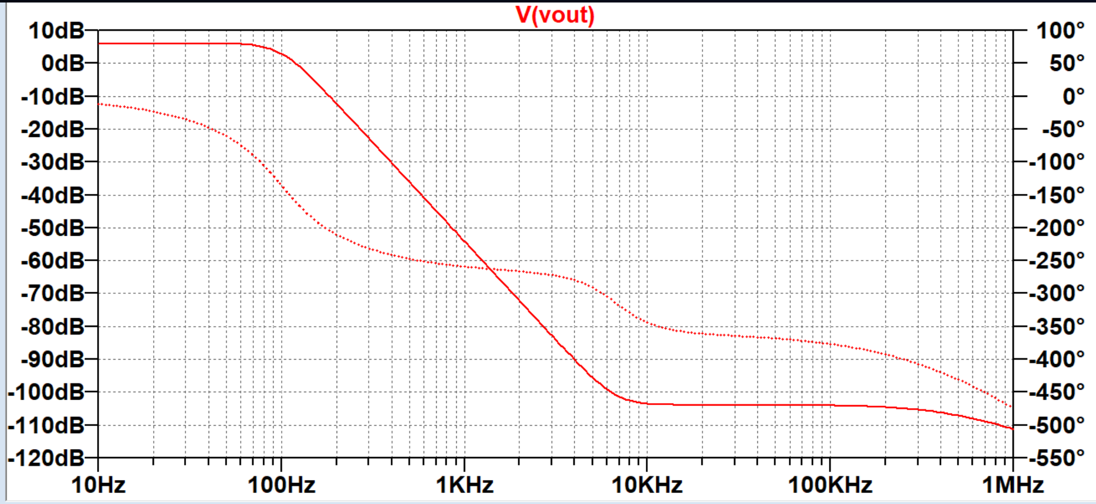
- Named after the British engineer, Stephen Butterworth, who developed a mathematical formula in the 1930s for designing a filter that has a flat frequency response in its passband
- A type of electronic filter used to selectively pass certain frequencies while blocking others
- Known for its flat frequency response within the passband, making it ideal for applications where a uniform signal amplification or attenuation is needed
- The filter's response is characterized by its order, which indicates the rate at which the filter attenuates signals outside of the passband.

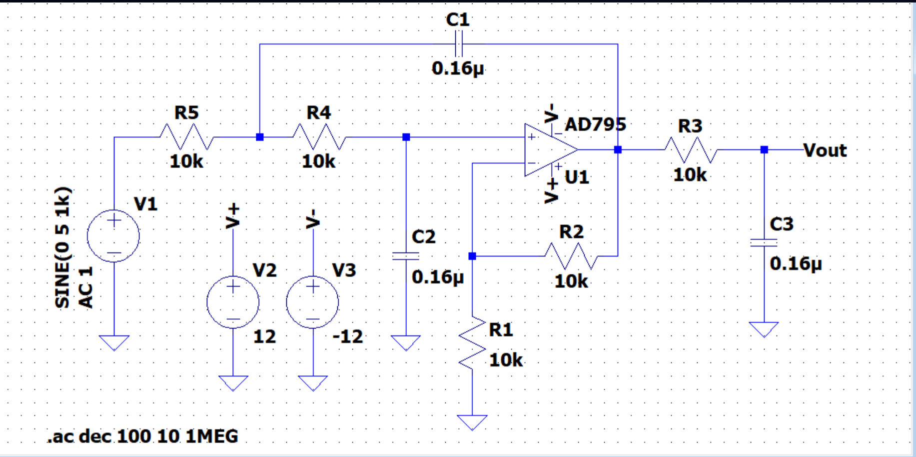
Design goals and advantages over normal filters

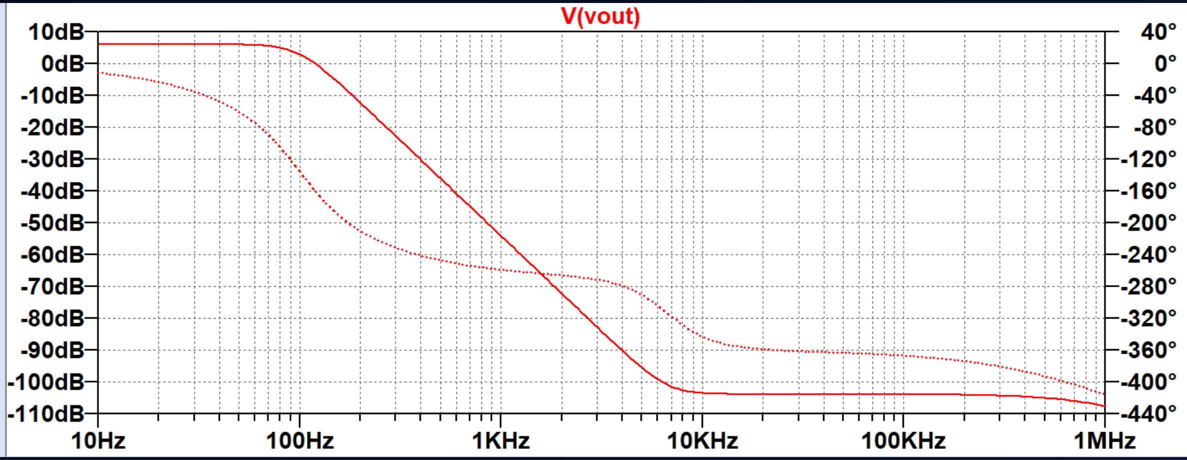
- Flat frequency response in the passband and attenuates unwanted frequencies evenly while normal filters may have non-uniform attenuation
- Preserves the shape of the original signal as much as possible (minimal ripples)
- Smooth roll-off in the stopband which gradually attenuates high-frequency signals (instead of sudden changes) to avoid phase shifts or distortions to the signal
- Applications: Audio and image processing to remove unwanted noise or enhance certain images, data converter applications as an anti-aliasing filter, and communication & control systems.

General third order Butterworth filter designs









Filter design task

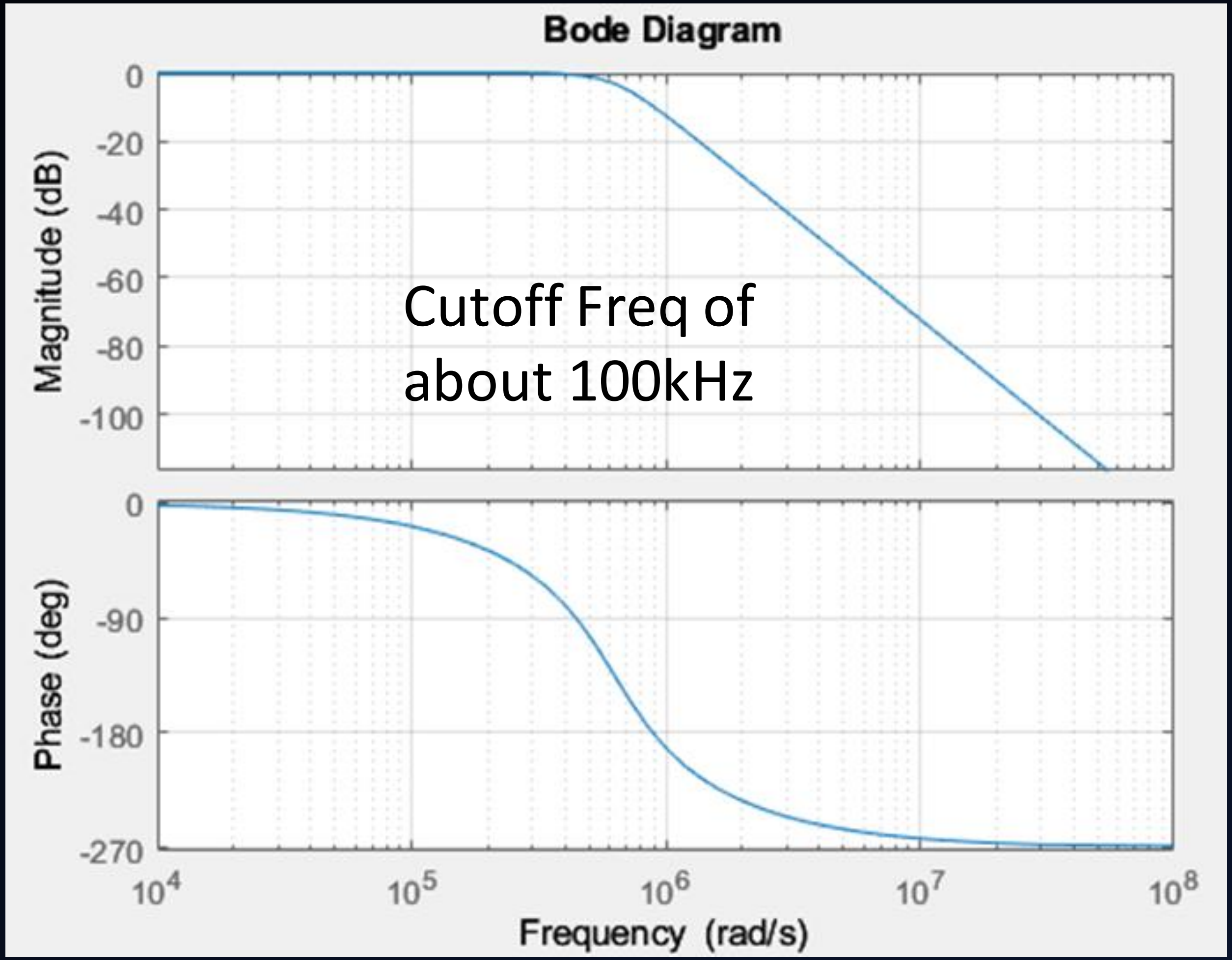
- To design a filter that has a transfer function H(s):

$$H(s) = \frac{a_0}{s^3 + a_2 s^2 + a_1 s + a_0}$$

- Where coefficients are as follows:

$$a_2 = 1.257 \times 10^6$$
$$a_1 = 789.6 \times 10^9$$
$$a_0 = 248.1 \times 10^{15}$$

Expected Magnitude Response



Expected output expression

$$H(s) = \frac{a_0}{s^3 + a_2 s^2 + a_1 s + a_0} = \frac{Y(s)}{X(s)}$$

$$\left[Y(s)s^3 + Y(s)a_2 s^2 + Y(s)a_1 s + Y(s)a_0 = X(s)a_0\right]\frac{1}{s^3}$$

$$Y(s) + \frac{Y(s)a_2}{s} + \frac{Y(s)a_1}{s^2} + \frac{Y(s)a_0}{s^3} = \frac{X(s)a_0}{s^3}$$

$$y(t) + a_2\int y(t) + a_1\left(\int\right)^2 y(t) + a_0\left(\int\right)^3 y(t) = a_0\left(\int\right)^3 x(t)$$

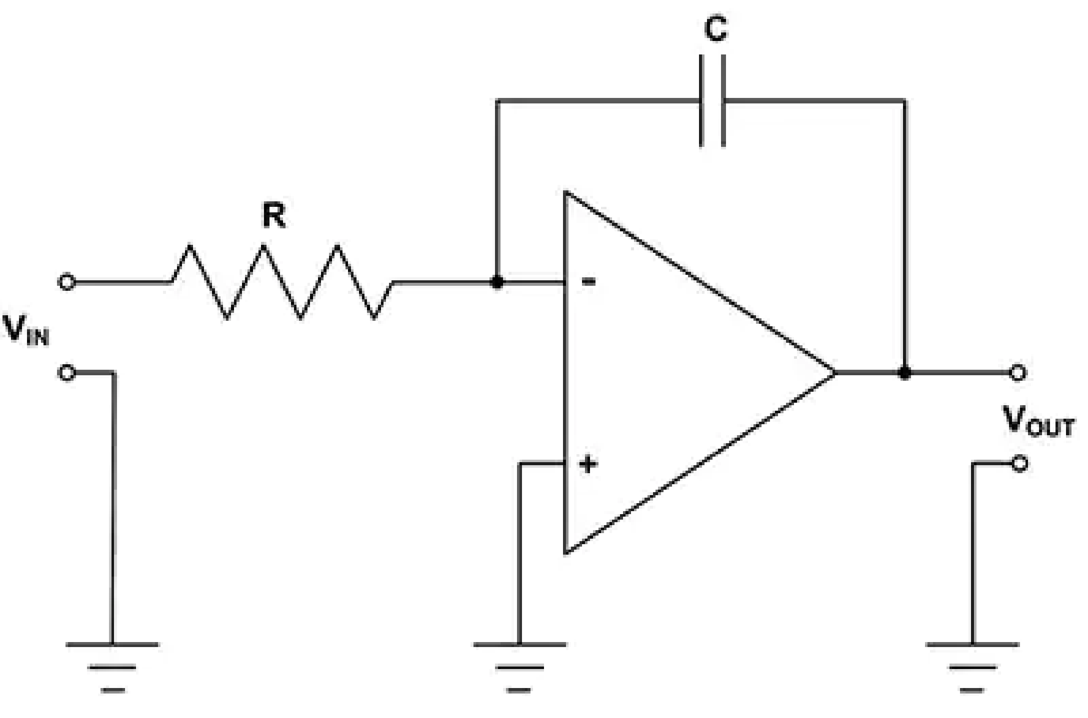
$$y(t) = a_0\left(\int\right)^3 x(t) - a_2\int y(t) - a_1\left(\int\right)^2 y(t) - a_0\left(\int\right)^3 y(t)$$

Topologies Involved

- For this project we decided to implement this filter in a rather non-traditional manner using active difference-integrators using resistors and capacitors (similar to Sallen-Key filter)

Topologies: Inverting integrator

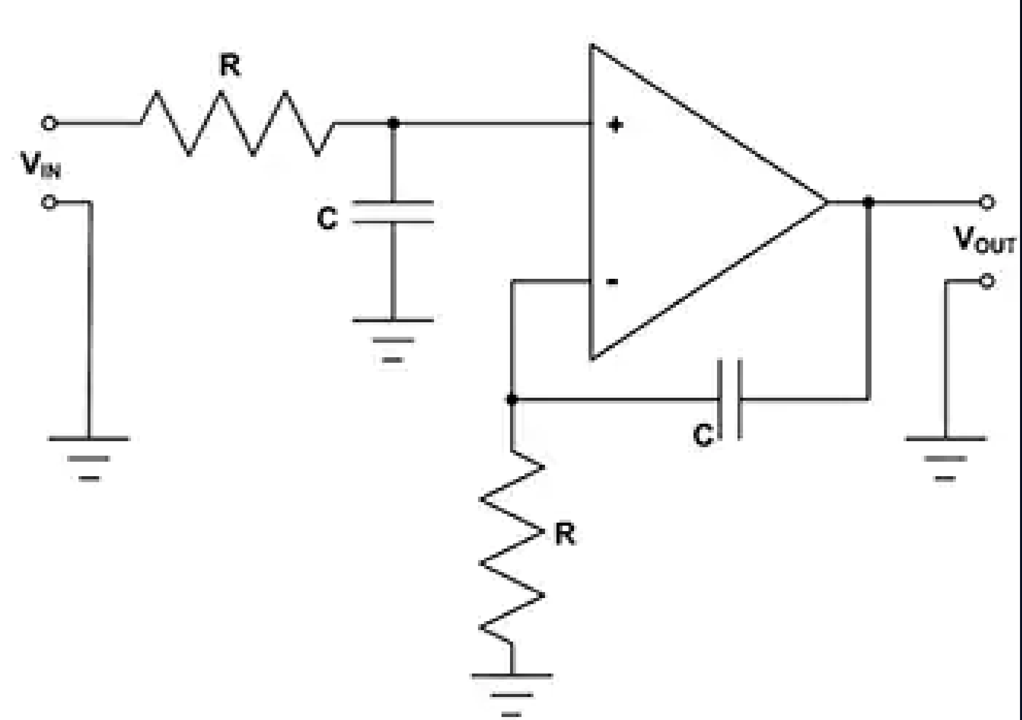
- The standard op-amp integrator topology is the inverting integrator, which outputs the integral of the input with a gain of –1/RC
- Useful for the y(t) integrals, useless for the x(t) integral



$$V_{OUT} = -\frac{1}{RC}\int V_{IN}dt$$

Topologies: Non-Inverting integrator

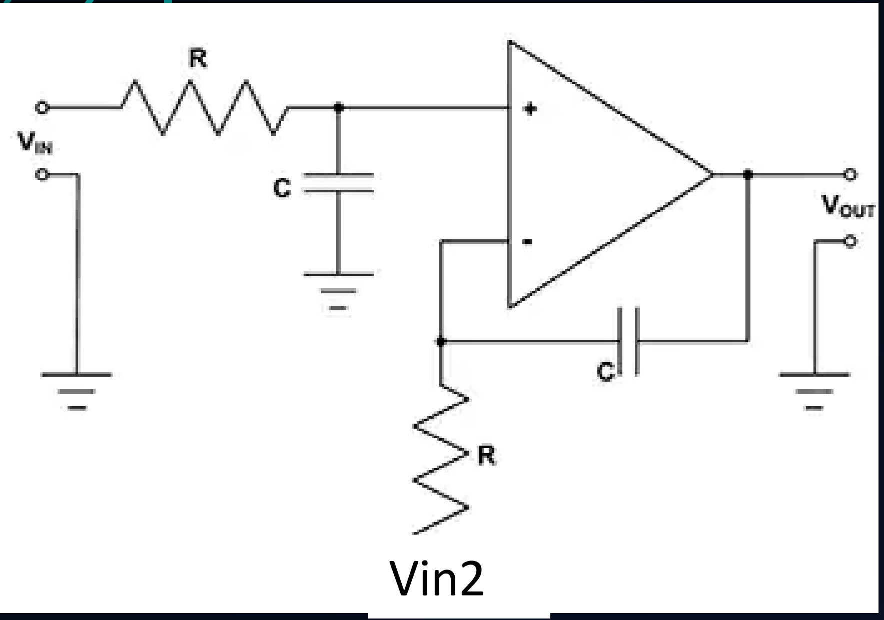
- Similar to the inverting integrator, except it outputs the integral of the input with a gain of +1/RC
- Useful for the x(t) integral, useless for the y(t) integrals



$$V_{OUT} = \frac{1}{RC}\int V_{IN}dt$$

Topologies: Difference integrator

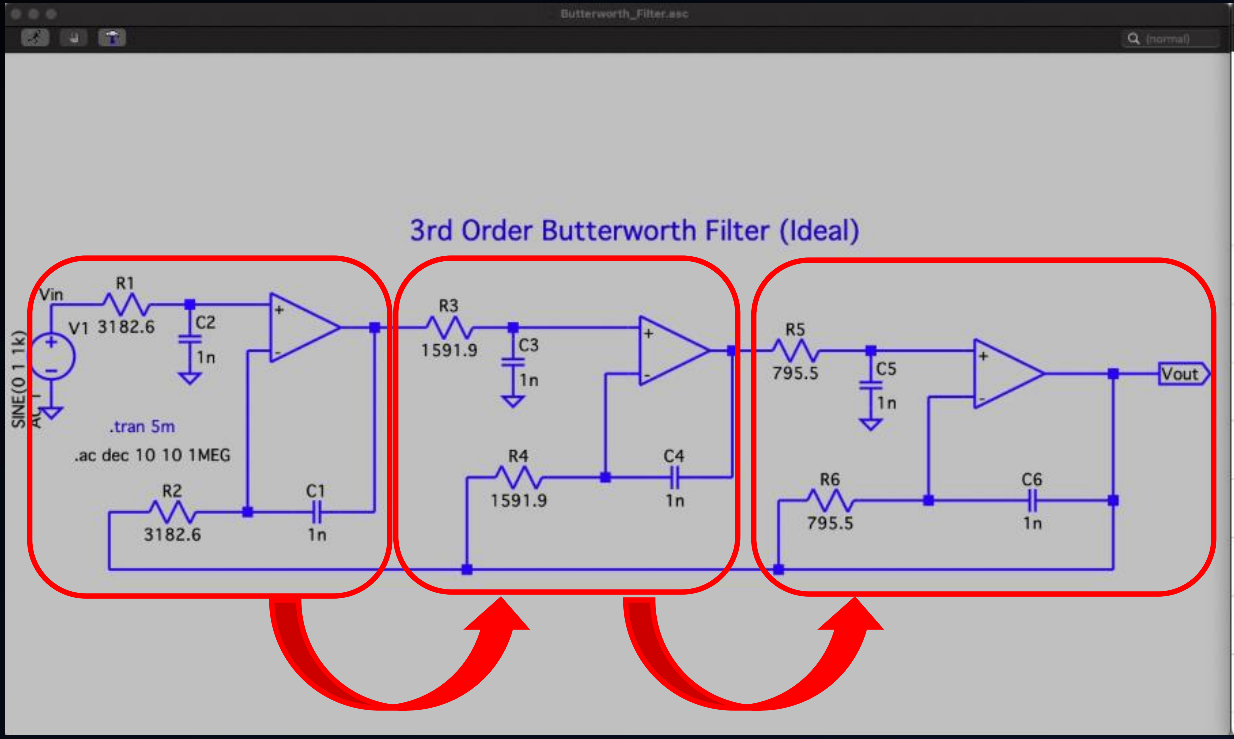
- Realizing that the Non-Inverting integrator is basically a difference amplifier/subtractor with some resistors replaced with capacitors and one input grounded, we can deduce that a difference integrator will take the integral of the difference between two inputs with a gain of 1/RC

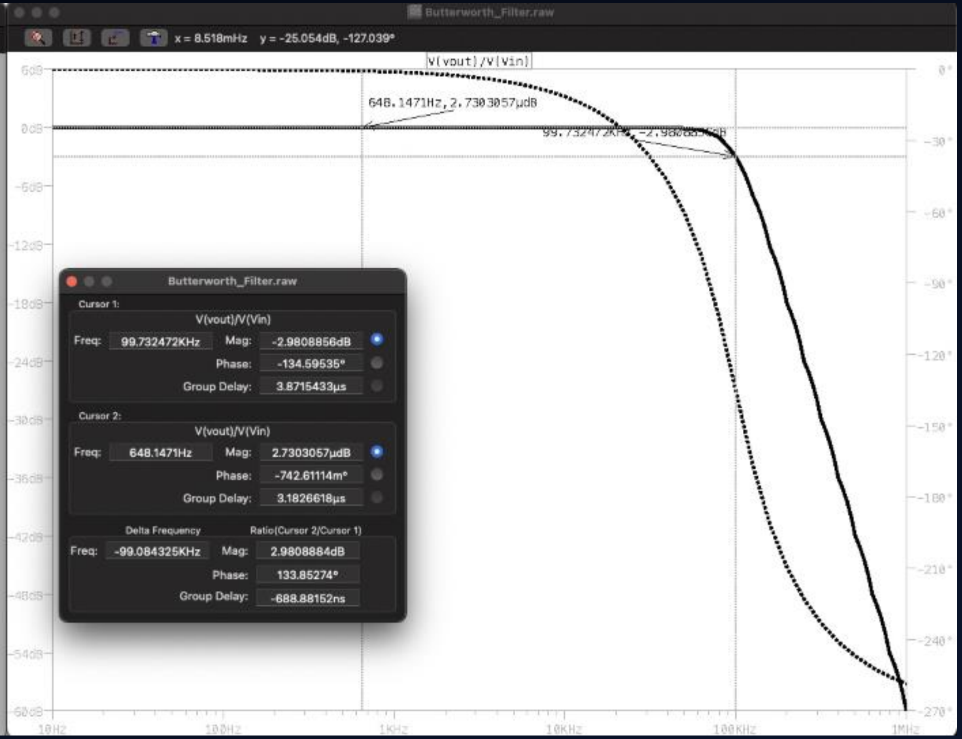


- Useful for all terms of output expression
- Will need total of three for triple integrals

$$Vout = \frac{1}{RC}\int Vin1 - Vin2$$

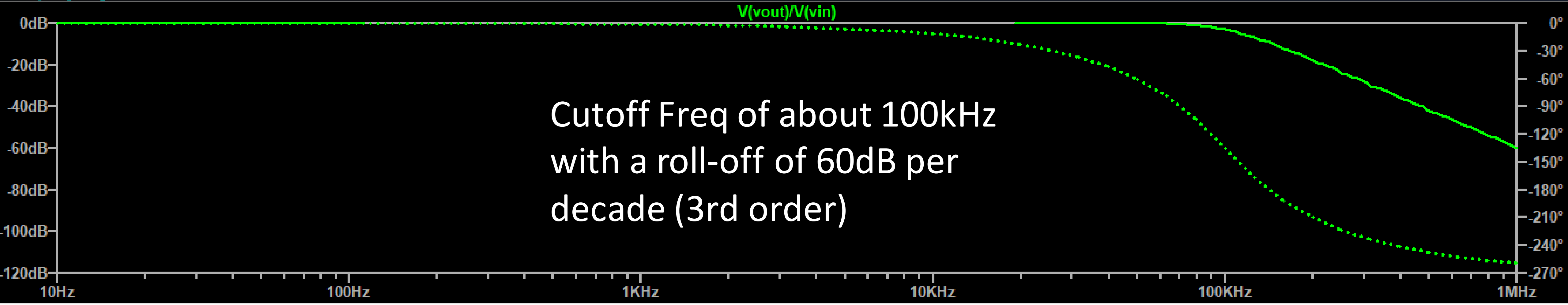
Circuit Design (using RC integrators)



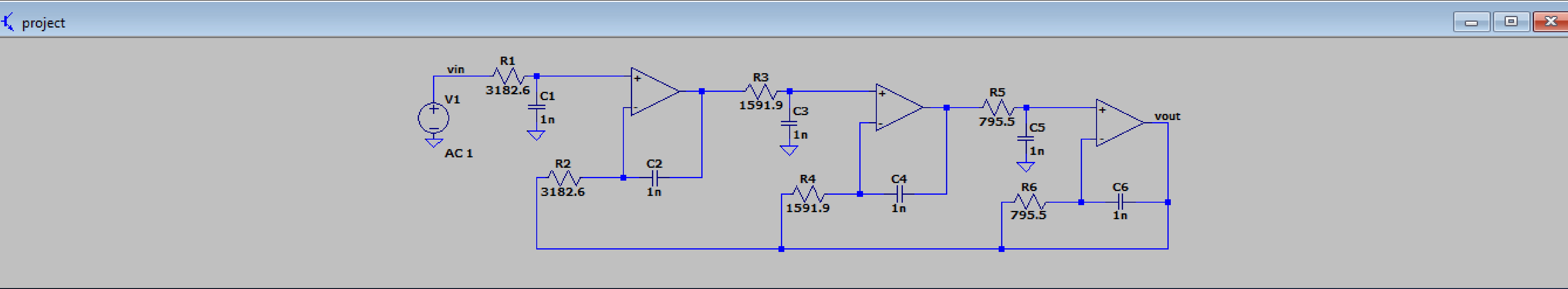


In order to acquire correct gain coefficients, the 1/RC integrator gains of each section is amplified the sections that follow it. Keeping this in mind, the gains can be determined through the selection of resistors and capacitors while also keeping values realistic (must be close to actual component values to build circuit in practice)

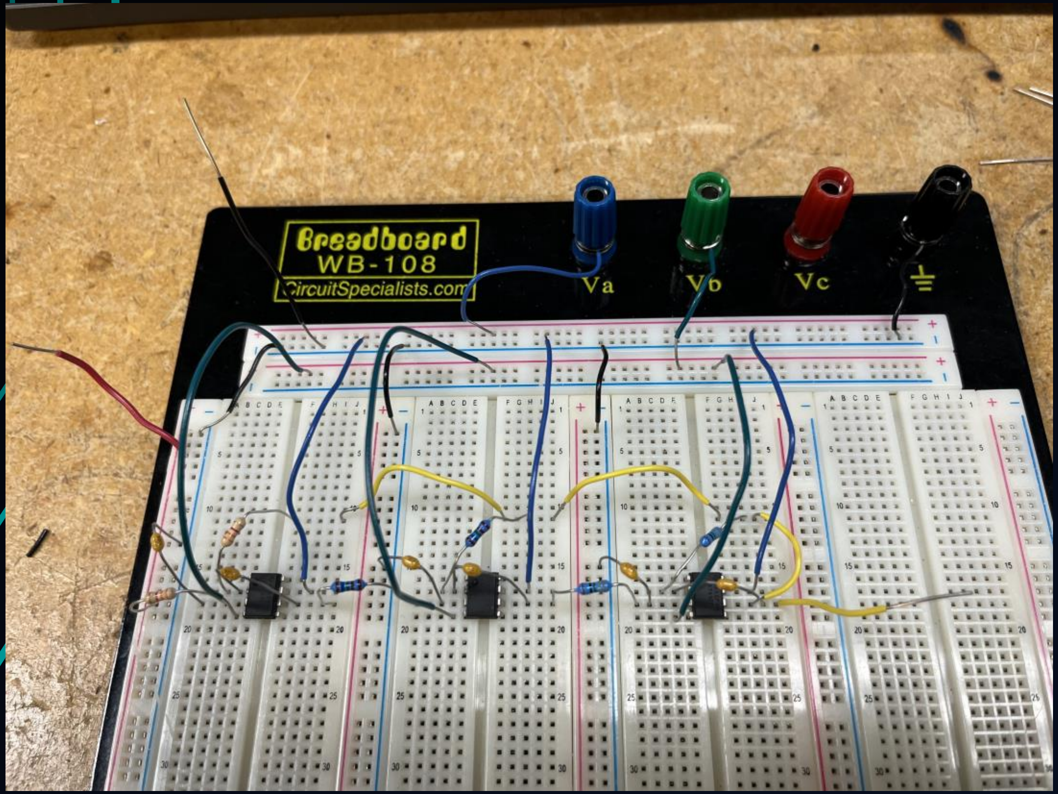
Bode plot simulation of design (Matches MATLAB)



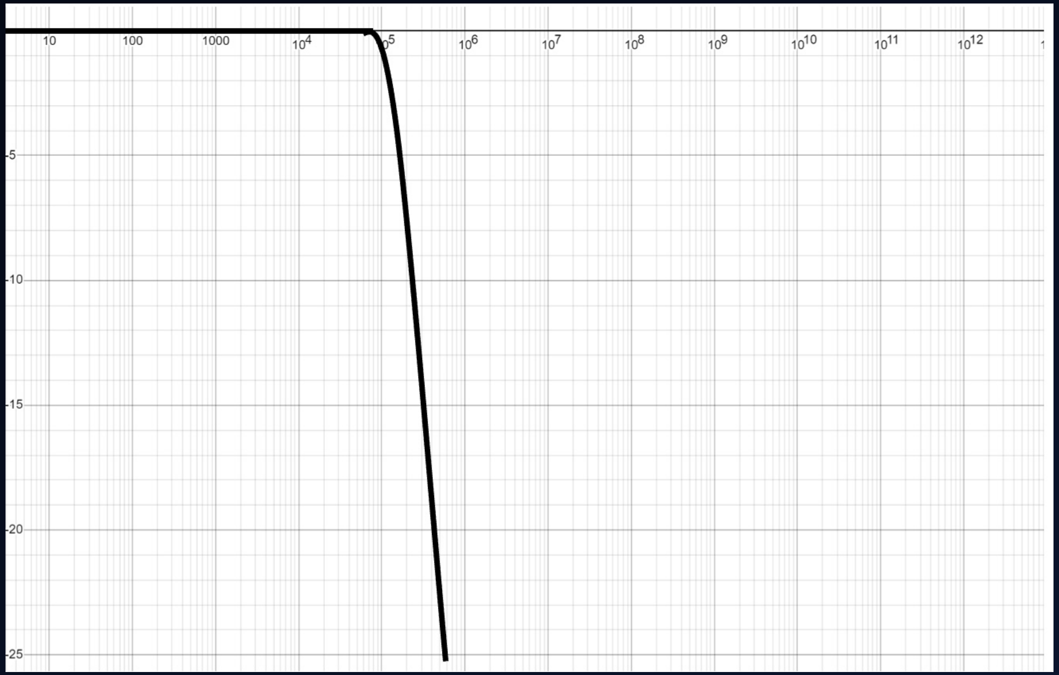
Cutoff Freq of about 100kHz with a roll-off of 60dB per decade (3rd order)



Circuit Design on the breadboard



x_1	y_1
10	0
100	0
1000	0
10000	0
100000	−0.935
120000	−4.86
140000	−10.10
160000	−11.53
250000	−19.8
500000	−24.3
1000000	−24.3
10000000	−24.3



Conclusions

- Designed to have a maximally flat amplitude response until the cut off frequency with a smooth roll-off
- Multiple ways of creating Butterworth Filters
- Achieved a maximally flat amplitude using a non-traditional method using a difference-integrator op-amp topology
- The only problem encountered was the design process being somewhat meticulous since the gain of the subsequent stages increases the gain of the previous stages.